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ABSTRACT

This volume presents four papers originally drafted for a symposium on sex differences and mathematics education held at the 1974 meeting of the American Educational Research Association. Subsequent to the AERA meeting the papers were revised. The paper by Fox reviews results of several contests to identify junior high school students who were precocious in mathematical ability, and subsequent instructional experiments aimed at improving the mathematical achievement of able girls. Aiken's paper presents factor analytic data concerning sex differences in attitudes toward mathematics and discusses several hypotheses to explain these differences. Armstrong's paper discusses results of factor analytic studies of sex differences in mathematics achievement and intelligence. Pennema's paper focuses on the role of spatial ability in learning mathematics and the relationship of this ability to sex differences in mathematics achievement. (SD)

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MATHEMATICS EDUCATION REPORTS

Mathematics Learning: What
Research Says About Sex Differences

edited by
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and Environmental Education
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Mathematics Education Reports

Mathematics Education Reports are being developed to disseminate information concerning mathematics education documents analyzed at the ERIC Information Analysis Center for Science, Mathematics, and Environmental Education. These reports fall into three broad categories. Research reviews summarize and analyze recent research in specific areas of mathematics education. Resource guides identify and analyze materials and references for use by mathematics teachers at all levels. Special bibliographies announce the availability of documents and review the literature in selected interest areas of mathematics education. Reports in each of these categories may also be targeted for specific sub-populations of the mathematics education community. Priorities for the development of future Mathematics Education Reports are established by the advisory board of the Center, in cooperation with the National Council of Teachers of Mathematics, the Special Interest Group for Research in Mathematics Education, the Conference Board of the Mathematical Sciences, and other professional groups in mathematics education. Individual comments on past Reports and suggestions for future Reports are always welcomed by the editor.

The recent release of achievement data in reading and mathematics by the National Assessment of Educational Progress has given a new and current emphasis to the problem of sex differences in school achievement. This problem has been "out of fashion" with researchers in mathematics education for several years. The new NAEP data clearly shows that ignoring the problem has not made it go away.

The genesis of this collection of papers was a symposium presented at the American Educational Research Association Annual meeting in Chicago in 1974. The symposium was jointly sponsored by two AERA Special Interest Groups: Research on Women and Education and Research on Mathematics Education. After the symposium, participants were asked to revise and extend their presentations in a series of papers. This publication is a result of that work.

The papers examine the research background on sex differences in mathematics achievement. Hopefully they will be the vanguard of additional research to come in this important area.

Jon L. Higgins
ERIC Associate Director
for Mathematics Education

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* The symposium which formed the basis for this publication was jointly sponsored by two Special Interest Groups: Research on Women and Education and Research on Mathematics Education. Subsequent to the AERA meeting, the papers were revised. Jenny R. Armstrong's paper was substituted for one presented by Carol N. Jacklin.

INTRODUCTION

Mathematics educators have often believed that girls achieve at lower levels in mathematics than do boys. Although this belief has generated no action to improve the learning of mathematics by girls, careful reading of the literature of the past decade or so reveals that this belief is not valid in all situations. In 1974 Fennema concluded reviewing the literature that "no significant differences between boys' and girls' mathematics achievement were found before boys and girls entered elementary school or during early elementary years. In upper elementary and early high school years significant differences were not always apparent. However, when significant differences did appear they were more apt to be in the boys' favor when higher-level cognitive tasks were being measured and in the girls' favor when lower-level cognitive tasks were being measured." No conclusion could be reached concerning high school learners. (Fennema, 1974, 136-137.) Maccoby and Jacklin made a stronger statement when they concluded that one sex difference that is fairly well established is "that boys excel in mathematics ability." (Maccoby and Jacklin, 1974, 352.) While this conclusion appears to be based on an incomplete review of the literature and is ambiguous because no distinction is made between ability and achievement, at the very least such a strong statement points up the need for in-depth analysis of the learning of mathematics by girls. -

Even though no definite conclusion can be reached at this time about the comparative levels of learning mathematics by the sexes, it is clear that, starting at about the tenth grade and continuing throughout all post high school education, girls increasingly chose not to study mathematics. Economic as well as moral reasons compel mathematics educators to be concerned with this problem. If mathematics is important for boys, it is equally important for girls. However, before a cry for change in the mathematical education of girls can be made, it is important to take stock of what is known about mathematics learning by boys and girls. The papers that follow explore some areas that are known to effect the learning of mathematics. Some hypotheses are suggested that, when data is available, will provide some insight concerning comparative mathematics learning by the sexes, and why they are unequal in their studying of mathematics.

The main factors related to sex differences in the learning of mathematics which are emphasized in this set of papers are intellectual factors. Largely omitted is the large set of social/cultural factors which effect the learning of mathematics as it is related to the development of one's sex role identity. A discussion of these highly important factors was omitted because, while there is a large body of knowledge about factors which effect the sexes differentially in achievement motivation in general, there is a paucity of data which deal explicitly with the learning of mathematics and the development of sex role identity. The reader should not assume that the omission of these factors reflects the lack of their impact on the learning of mathematics. On the contrary, it is hoped that lack of discussion of such societal influences will stimulate studies that will give direct information on the learning of mathematics and sex role

MATHEMATICALLY PRECOCIOUS: MALE OR FEMALE?

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Typically what has been learned of precocious mathematical ability and achievement in childhood and adolescence has been gleaned from retrospective study of the lives of eminent persons. Several famous scientists, mathematicians and quantitatively-oriented philosophers such as Pascal, Leibnitz, and Gauss were reported to have been mathematically precocious children (Cox, 1926). Since far fewer women than men have achieved eminence in mathematics it is not surprising that there are few reports of genius and childhood precocity among women (Cox, 1926; Bell, 1937; McCurdy, 1957; Stanley, 1974a; Stern 1971). There has been no evidence, however, to suggest whether or not precocious development is indeed more rare among females than males or merely less visible.

Perhaps because of their assumed rarity cases of precocious intellectual development and educational achievement have not been well-researched. Not even the monumental longitudinal study of intellectual giftedness by Terman (1925) provides information concerning precocious mathematical talent and achievement among children designated as gifted by measures of global intelligence.

An ongoing study of mathematical precocity at the Johns Hopkins University offers some interesting insight into the question of sex differences in mathematical precocity. First it provides information concerning the existence of precocious mathematical reasoning ability among adolescents, and secondly it explores the question of how precocious achievement in mathematics can be fostered.

The Existence of Precocious Mathematical Reasoning Ability in Adolescents

The Study of Mathematically Precocious Youth (SMPY) began in the fall of 1971 to search for junior-high-school-age students who were precocious in mathematical reasoning ability as evidenced by very high scores (660-800) on the Scholastic Aptitude Test Mathematics (SAT-M). In order to discover these talented students, SMPY conducted a talent search in each of the years 1972, 1973, and 1974. The rationale for discovering precocity was to use difficult pre-college level tests and this is discussed in depth elsewhere (Stanley, Keating, and Fox, 1974). The results of each year of testing are summarized in the following sections.

The 1972 Contest

In March of 1972 seventh, eighth and young-in-grade ninth grade students in the greater Baltimore area who had scored at or above the 95th percentile on the numerical subtest of an in-grade standardized achievement test such as the Iowa Tests of Basic Skills were invited to participate in a contest. Three hundred ninety-six students (223 boys and 173 girls) accepted the challenge and took the SAT-M.

The results of the testing were startling. Twenty-two boys (about 10 percent of the male contestants) scored 660-790. This is better than the average Hopkins student scored as an eleventh or twelfth grader. Clearly, there are many mathematically precocious boys. The highest score for a girl, however, was 600. Although 44 percent of the contestants were girls, 19 percent of the boys scored higher than the highest scoring girl. The difference in points between the highest scoring boy and the girl was 190 points (Stanley, 1973; Keating, 1974).

The mean scores for boys and girls in the contest, by grade, are shown in Table 1. Since the number of young-in-grade ninth graders was small

Table 1

Mean Scores on SAT-M for Students, by Grade and Sex,
in the 1972 Talent Search.

		Number	Mean
8th and 9th Grade	Boys	133	524
	Girls	96	456
7th Grade	Boys	90	460
	Girls	77	423

their scores are reported with those of the eighth graders. The highest mean score for any group was 524 for eighth and ninth grade boys. Seventh grade boys had a mean score of 460 followed by eighth and ninth grade girls and seventh grade girls with mean scores of 456 and 423, respectively.

The 1973 Contest

In the winter of 1973 a second talent search was conducted. This time students were considered eligible for the contest if they had scored at or above the 98th percentile on an in-grade numerical subtest of a standardized test such as the Iowa Test of Basic Skills. Wider publicity helped to increase the total number of students who participated. There were 666 students in the contest (420 boys and 246 girls). The percentage of girls, however, dropped from almost a half (44 percent) in 1972 to just

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over a third (37 percent) in 1973. This decrease in participation by girls may have been due in part to the fact there were actually two¹ contests in 1973 - one for mathematics in January and one in the verbal area in February. Students in both contests took the SAT-M and SAT-V. Students were told they could enroll for either contest and be eligible for prizes in both. The total number of students in both contests was 953. There were 537 boys (56 percent) and 416 girls (44 percent).

The highest SAT-M score for a girl in the 1973 contests was 650, while two boys (one a seventh grader) attained scores of 800 (Stanley, 1973). Seven percent of the boys in the 1973 contests scored 660 or more. No girl did. The mean scores on SAT-M, by sex, grade, and contest entered, are shown in Table 2.

Table 2

Mean Scores on SAT-M for Students, by Grade and Sex,
and Contest Entered, in the 1973 Talent Searches

		Mathematics Contest		Verbal Contest		Combined	
		Number	Mean	Number	Mean	Number	Mean
8th and 9th Grade	Boys	285	551	65	490	350	540
	Girls	158	511	103	446	261	485
7th Grade	Boys	135	495	52	434	187	478
	Girls	88	440	67	396	155	421

¹ In 1972 the Study of Verbally Gifted Youth (SVGY) was begun at The Johns Hopkins University. Thus in the winter of 1973 there were two contests. SMPY held their contest in January and SVGY held theirs in February. The SAT-M and SAT-V were given at both contests. Students were told to register for the January contest if they were primarily interested in mathematics, and to register for the February contest if their interests were primarily in the verbal area. Students were eligible, however, for prizes in both contests.

In the total group of contestants for both contests eighth and ninth grade boys scored highest (540) followed by eighth and ninth grade girls (485) and seventh grade boys (478) and girls (421). Girls in the mathematics contest in both grades scored lower than the boys in their grade-group who came for the mathematics contest, but scored higher than either boys or girls in their grade-group who were tested in the verbal contest.

The 1974 Contest

In January of 1974 a third talent search for mathematics was held. Students throughout the entire State of Maryland who had scored at or above the 98th percentile on the numerical subtest of a standardized achievement test were eligible for the contest. The testing was conducted in four centers across the state (The Johns Hopkins University, University of Maryland at College Park, Salisbury State College and Frostburg State College).

A total of 1519 students took the SAT-M. Thirty-nine percent of the participants were girls (591).

Sixty-one students scored 660 or above. Seven of those students were girls. One girl scored 700. The highest score earned by a boy was 760. In 1974 less than 2 percent of the boys scored higher than the highest scoring girl. Mean SAT-M scores in 1974 are shown in Table 3, by grade and sex. The pattern of mean scores in 1974 was similar to that of 1973. There were sex differences within each grade-group in favor of the boys.

Table 3

Mean Scores on SAT-M for Students, by Grade and Sex,
in the 1974 Talent Search

		Number	Mean
8th, 9th, & 10th Grade	Boys	556	541
	Girls	369	503
7th Grade	Boys	372	473
	Girls	222	440

Sex Differences

Boys and girls who participated in a voluntary mathematics contest (and who qualified for that contest on the basis of high scores on standardized tests of grade-level mathematics achievement) differed considerably with respect to performance on a difficult pre-college level test of mathematical reasoning ability. Mean scores for boys in the contests have been at least 35 points higher than for girls in each of the three years.

Thus as early as grades seven and eight boys out-perform girls on difficult pre-college level tests of mathematical reasoning ability and the differences are particularly striking at the upper ends of the distributions. In three years of searching SMPY has identified considerably more males than females who are highly precocious mathematical reasoners. The self-selection aspect of a contest may have contributed to the greater male than female participation in the contest but this does not explain why the ratio of boys to girls who scores 660 or above (16 to 1) was so much greater than the overall ratio of boys to girls in the contests (1.4 to 1).

Whether or not these apparent differences in mathematical aptitude for the two sexes is a result of biological differences or differential cultural reinforcements over time, or a combination of the two, is not clear. One would expect to find a large gap at the upper end of the distribution of mathematical ability (as was found by SMPY) if the biological explanation of sex differences in mathematical ability is correct. At the present time, however, many researchers feel that there is too little known about the inheritance of specific abilities such as mathematical aptitude to justify such a conclusion (Maccoby and Jacklin, 1972; Astin, 1974).

Some researchers believe that the differences between the sexes in average performance on tests of specific abilities such as mathematics reflect differential cultural reinforcements over time which have shaped the career and educational goals, interests and achievements of the two sexes (Aiken, 1970; Astin, 1968a, 1968b, 1971; Hilton and Berglund, 1971). SMPY's study of the characteristics of mathematically precocious adolescents does lend some support for the social explanation of sex differences at the higher levels of ability and achievement.

Boys who scored 660 or more on SAT-M had stronger orientations towards investigative careers in mathematics and science and greater theoretical value orientations than did their less mathematically precocious male and female peers (Fox, 1973; Fox and Denham, 1974). Many of the highly mathematically precocious boys report studying mathematics and sometimes science textbooks systematically with the help of a parent or interested teacher, while others have worked informally with mathematical puzzles, games and books. What has motivated this extracurricular pursuit of knowledge appears to be strong theoretical and investigative values and interests.

Girls, even the most mathematically talented, are far less likely than boys, particularly the most mathematically talented boys, to seek out special experiences related to mathematics and science. Girls tend to have values and interests of a more social than theoretical nature (Fox, 1973; Fox and Denham, 1974). Thus it is not surprising that few girls report that they study mathematics on their own. Thus differential performance by the sexes on difficult pre-college level tests of mathematical reasoning ability at grades seven and eight could be partially a result of differential exposure to and practice with mathematical problem solving situations which result from different interests and value orientations.

Girls also appear to receive less encouragement at home to consider scientific pursuits. In a small sample of gifted students studied by Astin (1974) parents of boys often had noticed their sons' interest in science at an early age. Parents of boys typically reported that they had discussed college careers in science, mathematics, medicine, and engineering with their sons. These parents reported providing more scientific materials such as toys, books, and games for their sons than did parents of girls. Very few parents of girls had noticed their daughters' showing interest in mathematics or science at an early age. The occupations which these parents had discussed with their daughters were more apt to be traditionally feminine ones such as nursing and teaching. The parents of the girls had given less thought to future educational plans for their daughters than had parents of boys.

Fostering Precocious Achievement

Although it is difficult to draw conclusions about the relative influences of biological and social factors upon the performance on measures of aptitude (e.g., some would even argue the possibility that some of the differences in test performance are artifacts of biased test materials), there is clear evidence that precocious achievement in mathematics can be directly influenced by environmental factors. SMPY's attempts to foster acceleration in mathematics provide some interesting insight into the dynamics of precocious achievement among bright adolescent boys and girls.

SMPY has sponsored three experimental accelerated mathematics classes on the Hopkins campus and two classes in a public junior high school. The details of these classes are reported in detail elsewhere (Fox, 1974a, 1974b; George, 1974; Stanley, 1974b). A summary of the results of these five classes and their implications for understanding the differences between the sexes with respect to precocious achievement is presented in the following sections.

Class I - Boys and Girls

In the summer of 1972, 30 end-of-the-year sixth graders² (18 boys and 12 girls) were invited to a special summer mathematics class which met two hours a week. Fourteen boys (78 percent) and seven girls (58 percent) enrolled for the program. The initial success of the class in mastering Algebra I with only 18 hours of instruction was so great that the class continued to meet for two hours a week through the middle of the following summer. Of the 21 students who initially began the course, six boys (43 percent) and one girl (14 percent) completed the study of all their pre-calculus mathematics (Algebra I, Algebra II, Algebra III, Plane Geometry, Trigonometry and Analytic Geometry). Six of the boys took calculus the following year in a senior high school.

Class II - Boys and Girls

In the summer of 1973, 85 students (51 boys and 34 girls) who had participated in the 1973 talent search and who had scored at least 500 on SAT-M and 400 on SAT-V were invited to a summer accelerated mathematics class. Most of these students were eighth graders who had completed Algebra I. Twenty-two boys (43 percent) and nine girls (29 percent) enrolled. Fourteen boys (64 percent) and none of the girls completed all the pre-calculus mathematics by the middle of the following summer meeting only two hours a week during the school year and four hours a week during the second summer (George, 1974).

Although these classes were highly successful in promoting precocious achievement in mathematics among boys, they were both far less successful with girls. First, more boys than girls were eager to enroll in such a program. Secondly, girls who did enroll tended to drop out of the classes before their completion.

Interviews with the girls indicated that one major reason for dropping out was a reluctance to become accelerated in their placement in school. Many of the girls seemed to fear being labeled as different from their friends by virtue of becoming somewhat accelerated. Girls also reported that the class meetings were dull, and some made references to the boys in the classes as "little creeps." The overall reaction to the classes by the girls was that it was socially unappealing and might have negative social consequences for the girls in school.

It has been reported that even very bright girls often self-select themselves out of advanced mathematics classes in high school (Haven, 1972) and that few women ever pursue doctoral degrees in mathematics (e.g., in 1969 only seven percent of the doctoral degrees awarded in mathematics were

² Thirty students were invited. One was an end-of-the-year third grader, Another was an end-of-the-year eighth grader. The remaining students were end-of-the-year sixth graders.

earned by women (Bisconti and Astin, 1973)). Until this present study, however, it was not known that bright girls in junior high school would be far more reluctant than boys to participate in special accelerated mathematics programs and, especially to persist in them.

Class III - An All Girls Class

The results of testing values and interests of boys and girls in the 1973 contest suggested that even the most mathematically able girls were likely to prefer social to theoretical activities. In combination with the results of the first two accelerated mathematics classes this suggested that to interest girls in learning mathematics faster it would be important to consider the social aspects of a program. Thus in the spring of 1973 an all-girls accelerated Algebra I class was organized for seventh grade girls who had been in the 1973 contest and who had scored at least 370 on SAT-M (the average of female juniors in high school)³. The details of the program for girls are reported elsewhere (Fox, 1974b). In brief, the class was designed to appeal to the social interests of girls in a number of ways. It emphasized social cooperation rather than competition and was taught by a woman rather than a man. Men and women scientists and mathematicians spoke to the girls about exciting careers in mathematics and science (such as operations research, health statistics, and social science research) which deal with social problems as well as theoretical ones. This approach to an accelerated program was considerably more effective in recruiting girls. Of the 34 girls invited, 26 enrolled (76 percent). Eighteen girls (69 percent) completed the course. Not all girls, however, chose to accelerate their mathematics in school the following year and a few actually met with resistance from their schools to their acceleration. Eleven did take Algebra II the following year; 10 of these (38 percent) were considered to have been successfully accelerated.

The emphasis on the social interests of girls was moderately effective in promoting greater achievement in mathematics for girls than had the two mixed-sex more theoretically taught classes. This approach, however, did not promote the same extent of acceleration for the girls that the other two programs did for the boys. Five of the girls from the all-girl class have indicated some interest in becoming further accelerated in mathematics (by as much as two or three years) by the time they complete high school and enter college.

³Two girls who had not participated in the 1973 contest were later tested on SAT-M and allowed to take the course. One of these girls scored 350 on SAT-M. Since she had been eligible for the first class but had not enrolled, the decision was made to let her be in the all girls class. Her score of 350 was considered to be an under-estimate of her ability. The following year she scored 570 on a different form of the SAT-M.

Classes IV and V - City Public School

In the winter of 1974 Leon Lerner, a guidance counselor at Roland Park School in Baltimore City, asked SMPY to set up in that school a fast-paced mathematics class based on the principles learned from classes I-III. Twelve boys and twelve girls in grades four through seven were selected to participate. On the basis of past experience SMPY suggested that there be two fast-paced classes - one for boys taught by a male college professor and one for girls taught by a female college professor. One boy and one girl dropped out of the program. Both classes made rapid progress through Algebra I meeting two hours a week for a total of 37 hours the first year and all who remained in the school the following year elected to continue in the fast-paced class to study Algebra II. Although on an average the girls were a somewhat less able group than the boys, the two groups performed about equally well on a standardized Algebra I test at the end of the first year. Both classes were considerably more successful in mastering Algebra I than the class of eighth graders in a regular Algebra I program for a full year (Stanley, 1974b).

The success of these two classes in fostering high achievement at an accelerated pace suggests that special programs of this type may be more successful for girls when they are conducted within the context of the regular school. Further research is needed to determine just how successful these programs can become for both boys and girls if implemented on a large scale within public schools or school systems. Whether or not sex-segregation and women teachers as role models are actually crucial for the success of girls needs to be studied systematically within school settings.

Conclusions

On the basis of SMPY's research on the mathematically precocious, it appears that males are more likely than females to perform at a very high level on pre-college level tests of mathematical reasoning ability (at least in a voluntary contest situation). The sizable gap between the sexes on mean SAT-M scores and at the upper end of the distribution as early as grade seven suggests that there may be biologically-based differences between the sexes with respect to mathematical aptitude. There are, however, strong indications that some of the apparent differences are related to environmental factors. Whether or not greater efforts to encourage and develop mathematical interests among women in childhood and adolescence could eliminate or reduce this sex difference at the higher levels of ability is not known.

Clearly it is much more difficult to foster precocious achievement and acceleration in mathematics among girls than boys. Some attention to the social interests of young women in structuring learning environments to foster accelerated achievement appears to increase the rate of participation and success of females. To date, however, SMPY has not effectively helped to accelerate any girl as far or as fast as most of the boys in its programs. This should not be interpreted as meaning that it is unprofitable to work with bright girls. Although mathematical precocity (both in measured ability

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and achievement) is far more evident among young males than females, SMPY's efforts to foster greater achievement among very bright students does suggest that girls can be helped to develop their quantitative potentials more fully.

Even if there are biologically based differences between the sexes which account for much of the differing degree of precocity between the sexes, it is still desirable to develop ways of fostering greater achievement among women as well as men. It would appear, however, that our instructional strategies and classroom environments should be more systematically studied and regulated to avoid unnecessarily discouraging young women from developing their mathematical potentials to the fullest.

The fact that mathematical precocity appears to be rarer and not just less visible among females than males in adolescents at the present time can lead us into one of two directions for future educational planning and development. First, we could concentrate upon boys all efforts to find and foster high level achievement and talent in mathematics, since they will be easier to find and to work with. (This first direction sounds very much like what, perhaps unintentionally, is occurring in most schools today.) The second direction would be to concentrate our efforts to identify talented young women as well as young men but modify or restructure our instructional strategies for girls to optimize their chances for high level achievement. The long-term benefits of this second approach could have some quite gratifying results.

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SOME SPECULATIONS AND FINDINGS CONCERNING SEX DIFFERENCES IN MATHEMATICAL ABILITIES AND ATTITUDES

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The notion that boys and men are better in mathematics than girls and women, speaking both historically and currently, has received attention in various sources. Looking at the matter historically, Hypatia was apparently the only woman mathematician of importance prior to the 18th century. Since then Maria Agnesi, Sophie Germain, Sonja Kovalevsky, Emmy Noether, and perhaps a few others are noteworthy. But these are only a handful compared to the number of men who have achieved eminence in mathematics during the same period (Iacobacci, 1970). On the more mundane contemporary educational scene, the typical finding in the elementary school is that girls are at least equal to and sometimes better than boys in mathematics achievement, and that the mathematical interests of the two sexes are not significantly different. At the junior high level, however, boys begin to surpass girls in mathematical achievement and to express greater interest in the subject. The superiority of boys in mathematics after grade seven is revealed particularly on problem-solving tasks such as those appearing on mathematical reasoning tests (Hilton & Berglund, 1974; Jacobs, 1974; Jarvis, 1964).

Differentiation Hypothesis

There is evidence that the factor structure of mathematical ability becomes more differentiated with maturity (Very, 1967; Dye & Very, 1968). These studies also reveal a greater number and more sharply differentiated factors, especially in the mathematical reasoning and spatial areas, in high school males than females. The observation that the abilities of both boys and girls become more differentiated as the individual passes through the school grades and has a greater variety of experiences is consistent with Ferguson's (1956) hypothesis that particular abilities result from transfer of training among different tasks required of people in a given culture. Thus, the emphasis in Western culture on the acquisition by girls of relatively greater verbal skills, as opposed to quantitative reasoning and spatio-perceptual skills, may explain in part why a well-differentiated verbal factor but less distinct quantitative reasoning and spatial factors are often obtained when a variety of psychological tests are administered to girls.

Four Explanations of Sex Differences in Abilities

It is debatable whether sex differences in mathematical abilities are the cause or the effect of the popular conception that mathematics is primarily a masculine enterprise. In any event, at least four explanations have been proposed to account for these differences: the sex-linked, recessive gene hypothesis; the masculine-identification hypothesis; the same-sex role modeling hypothesis; and the differential social expectations and

reinforcement hypothesis. Stafford (1972) proposed that sex differences in numerical and spatial abilities are due in large measure to the fact that these abilities - like pattern baldness, hemophilia, and red-green color blindness - are transmitted by sex-linked, recessive genes. If this hypothesis is correct, then a person's mathematical ability should be more closely related to that of the opposite-sex parent than to that of the same-sex parent. Reported findings that mother-son and father-daughter correlations on measures of this ability are equal to each other but higher than the mother-daughter correlation, and that the father-son correlation is essentially zero have been cited in support of this hypothesis. Although this is a compelling and timely hypothesis, it has encountered certain technical difficulties (see Garron, 1970). An alternative hypothesis proposed by Weiss (1973) maintains that an autosomal-recessive allele in the homozygous state is a prerequisite for the ability to carry out highly qualified mathematical and technical work.

A second explanation - the masculine-identification hypothesis of Plank and Plank (1954) - states that since working mathematics is an aggressive, masculine occupation, both boys and girls who are good at it identify more closely with their fathers or other strong male figures. An implication of this hypothesis is that women mathematicians are more masculine than women non-mathematicians. Cited in support of the masculine-identification hypothesis are studies relating father absence to ability in mathematics (Carlsmith, 1964; Landy, Rosenberg, & Sutton-Smith, 1969). But the notion that a masculine interest pattern and a higher level of aggression are associated with proficiency in mathematics has failed to receive much empirical support. At the very least the evidence shows that women mathematics majors are not necessarily aggressive or "masculine."

Related to the masculine-identification hypothesis is the proposal that sex differences in mathematical ability are the consequence of same-sex role modeling (Bem & Bem, 1970; Maccoby, 1966; Milton, 1958). According to this explanation, since mothers are typically more verbally oriented and fathers more quantitatively oriented, young girls who model their behavior after their mothers come to view themselves as incompetent in mathematics. A fourth hypothesis maintains that sex differences in quantitative ability are produced by differential social expectations and reinforcement. According to this hypothesis, boys are expected to be more proficient in mathematics and receive more positive social reinforcement than girls for success in the subject. As a consequence of such expectations and encouragement, boys come to be better in mathematics.

Some research, much of it not very well designed, has been conducted to test the last two hypotheses and to find ways of improving girls' mathematical abilities. Two conclusions stemming from this research are that: (1) improving their attitudes toward mathematics seems to help girls do better in the subject; (2) casting mathematical problems in terms of "typical feminine interests" content helps improve girls' scores on those problems (Carey, 1958; Graf & Riddell, 1972; Milton, 1958).

Sex Differences in Attitudes

Although sex differences in attitudes and anxiety toward mathematics are not always found (Jacobs, 1974; McClure, 1971; Roberts, 1970), as with differences in mathematical achievement, attitude differences tend to favor boys after elementary school (Hilton & Berglund, 1974; Keesee, 1973; Nevin, 1973; Poffenberger, 1959; Simpson, 1974). It has also been reported that mathematics test anxiety is significantly higher for eighth grade girls than for eighth grade boys (Szetela, 1973).

Greater interest and more positive attitudes toward mathematics and science on the part of males have been found in other countries as well as the United States. For example, Nevin (1973) concluded that Irish girls have a deeper interest in human relationships than their male counterparts, which interferes with an interest in mathematics. In a discussion of the results of a study on mathematically precocious youth, Fox (1974) also suggests that the greater social interests of girls, especially during adolescence, may interfere with concentrated effort on mathematics.

Ability and attitude are, of course, not independent psychological dimensions, but rather are interrelated components of human personality. The relationship between attitude or interest and performance is reciprocal and dynamic, in that attitude or interest affects achievement and achievement in turn affects attitude and interest. Consequently, it is possible that lack of success in mathematics, which could easily lead to a negative attitude toward the subject, is due to some extent to a genetically-determined low aptitude for the subject. But since there are disparities among countries in the magnitude of the differences in attitudes of boys and girls, biological explanations appear to be insufficient. The influences of differing socio-cultural "expectations," same-sex role modeling, and reinforcement for success in mathematical endeavors must also be taken into account.

A Composite Hypothesis

In keeping with a more dynamic explanation of mathematical ability and attitude, it is proposed that both specific abilities and attitudes be recognized as learned response tendencies shaped by socio-cultural experiences impressed on a more general, genetically-determined temperamental and ability base. This composite interpretation views the several components of mathematical ability as acquired differentiations of general cognitive ability, such differentiation being the consequence of the expectations of "significant others," the modeling of their behavior, the pattern of reinforcement supplied by them, and the transfer of skills and knowledges from one mathematical task to another. Attitude and other affective variables, which interact with aptitude to influence proficiency in mathematics, are also conceptualized as learned response tendencies. Such a dynamic, interactive explanation has been the guiding principle of my research on psychological factors in learning mathematics. This research, which has been concerned with junior- and senior-high students and college undergraduates and graduates, has involved correlational, multiple regression, and factor analyses of scores on a wide range of affective and cognitive variables.

A Research Illustration

As an example of these studies, Table 1 contains the results of two orthogonal powered-vector factor analyses - one for 72 eighth-grade girls and a second for 84 eighth-grade boys. These analyses are based on five ability measures, scores on the Mathematics Attitude Scale, and scores on sixteen items of a specially designed biographical inventory. Two independent factors were obtained for girls and one factor for boys. Factor A in both cases is interpreted as "Student's Mathematical Ability," and Factor B for girls is "Father's Reported Mathematical Ability." Note that since Factors A and B are orthogonal, it may be concluded that for the females "Student's Mathematical Ability" was independent of "Father's Reported Mathematical Ability." In contrast, for males "Student's Mathematical Ability" was positively related to "Father's Reported Mathematical Ability." These findings are clearly inconsistent with both the "masculine identification" and "sex-linked recessive gene" hypotheses.

Of interest in this connection is Hill's (1967) intensive study of 35 seventh-grade boys and their parents. Hill found that it was the father, rather than the mother, whose expectations for his son's performance in mathematics were associated with positive attitudes on the part of the son. This was especially true when the father expressed warm feelings for the son and participated more in rearing him. Hill interpreted these findings as supporting the position of identification theorists that the father is the critical parent for the sex-role learning of the son, but a simple same-sex modeling hypothesis was not a sufficient explanation of all the data.

Finally, it can be determined from Table 1 that mathematical ability in these junior-high school girls is more closely related to their attitudes toward mathematics than in the boys. Behr (1973) and others have reported similar findings, in addition to the fact that mathematical ability test scores and mathematics marks are more closely related in girls than in boys. Although there is some variation in predictability with age, using both affective and cognitive variables we are usually able to do a better job of forecasting the school mathematics grades of females than those of males. Consequently, another noteworthy sex difference - seemingly the reverse of the traditional stereotype - emerges from these studies. Women, at least on psychological and educational measures, are apparently more predictable than men!

Table 1
Factor Analyses of Eighth Grade Data

Variable	Girls (N = 72)		Boys (N = 84)
	Factor A	Factor B	Factor A
Eighth-grade final math mark	.791	-.025	.745
CTMM Language I.Q.	.826	.001	.773
CTMM Nonlanguage I.Q.	.774	.015	.728
CAT Arithmetic Reasoning	.922	.003	.897
CAT Arithmetic Fundamentals	.875	-.027	.862
Mathematics Attitude	.378	.061	.268
"My father and I have always been very close to each other."	.085	.092	.120
"I have an older brother who likes mathematics."	-.039	.164	-.330
"I have an older brother."	-.091	.013	-.337
"I have a younger sister."	.130	.058	-.030
"I have a younger sister who likes arithmetic."	.069	-.091	-.221
"My father graduated from high school."	.207	.270	.428
"My mathematics teachers in school have usually been somewhat impatient and demanding."	-.423	.030	-.285
"My father uses mathematics on his job."	.192	.146	.177
"My mother graduated from high school."	.396	.331	.385
"I have usually been an excellent student in school."	.169	-.114	.300
"My father made high grades in mathematics when he was in school."	.034	.828	.262
"In elementary school, my grades in arithmetic were usually high."	-.027	.122	.343
"My father is a professional man (doctor, lawyer, engineer, teacher, etc.)."	.299	.089	.393
"My mother made high grades in mathematics when she was in school."	.073	.154	.108
"My father likes mathematics."	.126	.822	.309
"I usually stick to a job until it is finished."	.088	-.110	.156

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FACTORS IN INTELLIGENCE AND MATHEMATICAL ABILITY WHICH MAY ACCOUNT FOR DIFFERENCES IN MATHEMATICS ACHIEVEMENT BETWEEN THE SEXES

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Introduction

Intelligence and mathematical ability, both of which are implied from the standardized observation of performance, are comprised of many of the same factors (see Figure 1). The nature of the relation between mathematical ability and intelligence, which will be explored in more detail, is important to any explanation of sex differences in mathematics achievement because of the assumed relation between mathematical ability, intelligence and mathematics achievement.

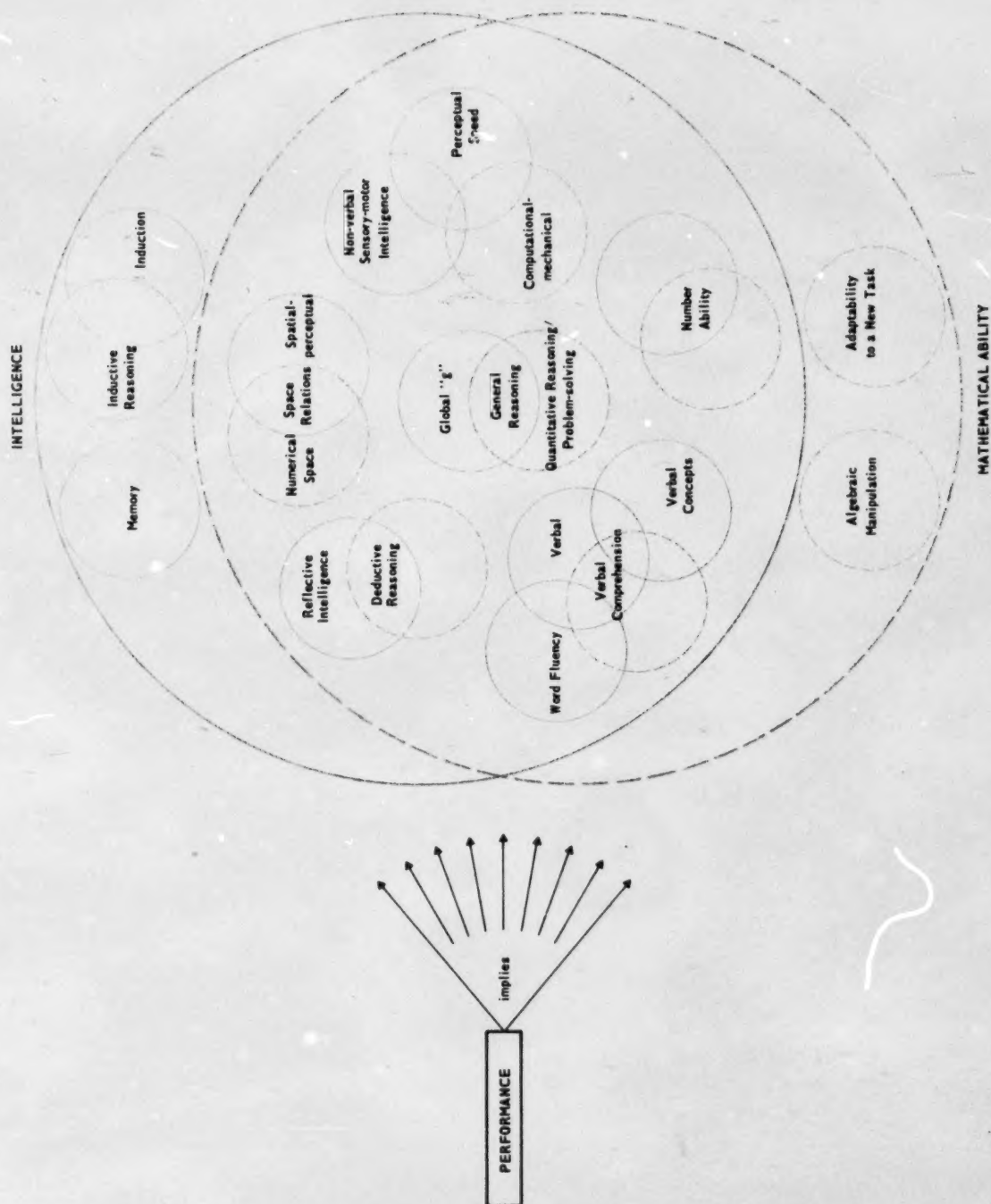
Intelligence, Mathematical Ability, and Mathematics Achievement

The existence of a strong, positive relation between intelligence and mathematics achievement and between mathematical ability and mathematics achievement has been consistently supported (Aiken, 1971). The identification of particular factors of intelligence and mathematical ability which are important to mathematics achievement has been less consistent. Similarly, the relation between factors of intelligence and mathematical ability has not been sufficiently clarified. Even so, there has been some work completed which provides insight into these relations.

The global "g" factor in intelligence has been found to be strongly related to general quantitative reasoning or problem solving in mathematical ability (Barakat, 1951; McAllister, 1951), and a sensori-motor factor in intelligence has been found to be strongly related to a computational-mechanical factor in mathematical ability (Coleman, 1956). In addition, a reflective intelligence factor has been found to be strongly related to a deductive reasoning factor in mathematical ability (Skemp, 1971). Memory has not been found to be an important factor and spatial ability only in a limited area of mathematics (Coleman, 1956).

Muscio (1962), in an examination of the relations among mathematical ability, achievement and intelligence, found strong positive relations between both mathematics ability and achievement and intelligence and mathematics achievement. Mathematical ability was also found to be related to certain reading factors, paragraph meaning, reading to understand precise directions, and reading to note details.

When intelligence was partialled out, the relation between mathematical ability and mathematical achievement, although decreased, was still significant. Specific factors in intelligence found to be related to



mathematical ability were: spatial relationships¹, numerical reasoning, logical reasoning, verbal concepts, total verbal, and total non-verbal (Muscio, 1962). The arithmetic achievement subtests which were found to be related to general intelligence were: arithmetic reasoning, arithmetic computation and mathematical vocabulary.

Other specific mathematical ability factors have been identified but the relation to intelligence is not clear. In one study, the following factors were identified: verbal comprehension, deductive reasoning, algebraic manipulative skill, number ability and adaptability to a new task (Kline, 1960). In another study, these factors were identified: algebra achievement, mathematical aptitude, verbal and number (Wooldridge, 1964). Yet another study identified the factors: general reasoning, deductive reasoning, numerical space and verbal comprehension (Werdelin, 1966).

Many of these same factors were identified in a factor study of intelligence by Harris and Harris (1971b). This study examines four models of intelligence: (a) Thurstones' (1938, 1941, 1944, 1962) Primary Mental Abilities Model; (b) Guilford's (1967) Structure of Intellect Model; (c) Guttman's (1970) Behavioral Model; and (d) Harris and Harris's (1972) Cognitive Model.

Using the methods for determining Common Comparable Factors (CCFs) (Harris & Harris, 1971a), the results of the study indicated that the most viable model for explaining the factorial structure of intelligence was the Thurstones' model (1938, 1941, 1944, 1962). The factors included in their model were: (a) spatial, (b) perceptual speed, (c) numerical, (d) verbal, (e) word fluency, (f) memory, (g) induction, (h) deduction, and (i) closure. Harris and Harris (1971b) found six of these nine factors represented for boys and five of these factors represented for girls. The five factors which were found to be the same for boys and girls were: (a) verbal, (b) induction, (c) memory, (d) word fluency, and (e) perceptual speed. A number factor was found for boys, but not for girls.

Similar to the findings of Harris and Harris (1971b), Aiken (1973), by summarizing the factor analytic work of several different investigators, identified several specific intelligence factors which have been found to be strongly related to both mathematical ability and achievement. They were: deductive (general) reasoning (Blackwell, 1940; Kline, 1960; Very, 1967; Werdelin, 1958, 1966), inductive reasoning (Werdelin, 1958), numerical ability (Kline, 1960; Very, 1967; Werdelin, 1958, 1966; Wooldridge, 1964), spatial-perceptual ability (Blackwell, 1940; Very, 1967; Werdelin, 1966), and verbal comprehension (Blackwell, 1940; Kline, 1960; Very, 1967; Werdelin, 1958, 1966; Wooldridge, 1964).

¹ The relations between spatial ability and mathematics achievement are examined and discussed in more detail in this paper series by Fennema.

Sex Differences in Intelligence, Mathematical Ability and Achievement

Sex has been included as a variable in several studies of mathematics achievement which were reviewed by Fennema (1974). The conclusions drawn about the nature of sex differences in mathematical achievement were: (a) there are no consistent differences in the knowledge of mathematics between boys and girls before they enter school or at kindergarten level; (b) there are no consistent differences in the learning of mathematics by boys and girls in the early elementary years; (c) there are no differences that consistently appear between the learning of boys and girls in mathematics in the fourth through ninth grades; there are, however, trends which indicate that if there are differences, and they occur during the upper elementary, junior high school or early high school years, girls tend to perform better on tests of mathematical computation, while boys tend to perform better on tests of mathematics reasoning (i.e., comprehension, application, and analysis); (d) there are no conclusions which can be drawn from the studies which have examined high school level subjects (Fennema, 1974).

Sex differences in intelligence and mathematical ability show a similar trend to mathematics achievement relative to emergence. In a review of research by Maccoby and Jacklin (1972, 1974), sex differences in intelligence and mathematical ability were not consistently found until early adolescence. Consistent differences between the sexes were found only in verbal ability and analytic ability. Girls and women tended to be superior in verbal ability and boys and men tended to be superior in analytic ability.

Differences between the sexes in analytic and verbal ability were also reflected in the factor analytic study of intelligence by Harris and Harris (1971b). Verbal items were found to consistently load higher on factors identified for the girls while analytic-type items tended to load higher on factors identified for the boys. As noted previously, a number factor was found for boys but not for girls. The types of items which loaded heavily to form the number factor for boys were: number class extension, seeing trends, arithmetic problems, and number relations. Number class extension, seeing trends, and number relations items all require analysis of a situation to determine the rule or generalization and then the specification or identification of examples which satisfy the rule. These items did not load on any factor for the girls. The remaining types of items on the number factor, arithmetic problems loaded on the word fluency factor for girls. As alluded to by French (1965) and Fredricksen (1969), this type of factorial composition may reflect the different cognitive strategies that boys and girls are using to solve the same problems. It would appear that boys are relying more on their analytic ability while girls are utilizing their verbal ability.

Whether the number factor identified by Harris and Harris (1971b) is the same or similar in content to number factors identified in other studies (Kline, 1960; Very, 1967; Werdelin, 1958, 1966; Wooldridge, 1964) is unclear. Canisia (1962), however, in a study of eleventh grade girls found the number factor identified by Kline (1960), Wooldridge (1964) and Werdelin (1966) to be unrelated to other mathematical ability factors. If the number factor

being assessed in these different studies is the same or similar, then the lack of relation found by Capisia (1962) may parallel the findings of Harris and Harris (1971b). Certainly more study of the nature of the factorial structure of intelligence in the sexes at different age levels needs to be done. This is particularly true due to the large difference in the ages of the girls in the Harris and Harris (1971b) and Canisia (1962) studies (i.e., fifth and eleventh graders).

Stafford (1972) in examining an hypothesis relating to sex-linked chromosomal transfer of "quantitative reasoning" ability, combined two of the types of items used in the Harris and Harris (1971b) study: (a) Necessary Arithmetic Operations and (b) Arithmetic Problems. Necessary Arithmetic Operations items, which were the original arithmetic reasoning types of items used by Thurstone and which more recently have been used as a part of the reasoning test from the NLSMA Reports (1968), are problems of the form:

Jane's father was 26 years old when she was born. Jane is now 8 years old. How old is her father now?

- A. subtract
- B. divide
- C. add
- D. multiply

"Arithmetic Problems"-type items involve computation. In the Stafford (1972) study, the items were combined. In order to get a correct score, the subjects in the study had to both identify the appropriate operation (Necessary Arithmetic Operations) and do the computation (Arithmetic Problems). The fact that these two types of items which loaded on separate factors in the Harris and Harris (1971b) study, were combined in the Stafford (1972) study make the results very difficult to interpret. Similarly, although Stafford (1972) suggests that there are sex differences in performance at each level (ages 12 to 18) favoring boys, no specific tests of significance were reported.

Necessary Arithmetic Operations items were also included as reasoning items in the NLSMA (1968) study and, as reported by Fennema (1974), trends indicated that boys in grades nine and ten were outperforming girls in the reasoning area. Necessary Arithmetic Operations items in the Harris and Harris (1971b) study did not load consistently and strongly on any factor. Necessary Arithmetic Operations did, however, show loadings on different factors for boys and girls. Necessary Arithmetic Operations items loaded on the verbal factor for boys and the induction factor for girls. Again, the differential loadings may imply different cognitive strategies for solving the same problems. In this case, however, it would appear that boys are using a weak ability (verbal) to solve a type of problem they have been found to perform well on elsewhere (NLSMA, 1968). Perhaps the age differences between the subjects in the two studies would account for these discrepancies. The subjects in the Harris and Harris (1971b) study were 10 and 11. The emergence of differences between the sexes on reasoning items in the NLSMA (1968) data are not well established for subjects until they were in the 14-15 year age range (Fennema, 1974).

Summary

In general, the results of studies of the relations among intelligence, mathematical ability and mathematics achievement indicate that there is a strong positive relationship both between intelligence and mathematical achievement and mathematical ability and achievement. Intelligence and mathematical ability have several factors in common. The nature of the relation between intelligence and mathematical ability is such that some of the same factors (or abilities) which have been defined as components in intelligence have also been defined as components in mathematical ability (see Figure 1). The specific factors which have been most frequently identified as components in both intelligence and mathematical ability are: (a) general reasoning, (b) deductive reasoning, (c) verbal comprehension, (d) general non-verbal factors, (e) space relations, and (f) number ability. Three factors have been identified in intelligence but not in mathematical ability: (a) memory, (b) inductive reasoning, and (c) induction. Two factors have been identified in mathematical ability and not in intelligence: (a) algebraic manipulation and (b) adaptability to a new task (see Figure 1).

General reasoning. General reasoning has been identified as a factor both in intelligence and mathematical ability. The general reasoning factor in mathematical ability has also been found to be related to the global "g" factor in intelligence which has in turn been found to be related to quantitative reasoning and problem solving in mathematical ability. General reasoning (i.e., global "g", quantitative reasoning and problem solving) have all been found to be related to mathematics achievement. No consistent sex differences have been found in general reasoning ability.

Deductive reasoning. Deductive reasoning has been identified as a separate factor in both intelligence and mathematical ability. The reflective intelligence factor has been found to be related to the deductive reasoning factor in mathematical ability. Deductive reasoning (logical reasoning) has been found to be related to mathematical achievement. No evidence is available to support differences between the sexes on these factors.

Verbal comprehension. Verbal comprehension has been identified as a factor in both intelligence and mathematical ability. The verbal concepts factor and the verbal factor of intelligence have been found to be related to the verbal comprehension factor in mathematical ability (Muscio, 1962). Word fluency was found to be a separate but related factor to the verbal factor in intelligence (Harris & Harris, 1971b). Verbal factors in intelligence and mathematical ability have been found to be related to mathematics achievement. Prior to adolescence, no consistent sex differences have been found in verbal ability. During early adolescence, however, sex differences in verbal ability emerge. Girls and women are found to be superior to boys and men in verbal ability.

General non-verbal factors. Sensori-motor intelligence has been found to be related to the computational-mechanical factor in mathematical ability. "Non-verbal" and "sensori-motor" are both terms which have been used to refer to the non-verbal items on standardized assessment tests. Perceptual speed

is a separate factor of intelligence which has been identified and which would seem to be related to both non-verbal intelligence and computational-mechanical factors. No research, however, has been reported to support that relation. Sensori-motor intelligence and computational-mechanical ability have both been found to be related to mathematical achievement. No sex differences have been found on these factors.

Space relations. Space relations have been identified as a factor in both intelligence and mathematical ability. Numerical space has been identified as a factor in mathematical ability and spatial-perceptual ability has been identified as a factor in intelligence. There is little evidence to support, however, any relation between mathematics achievement and spatial ability. Similarly, there is little evidence which suggests that there are consistent sex differences on this factor.

Number ability. Number ability has been found as a factor both in intelligence and mathematical ability. This factor has also been found to be related to mathematical achievement. This factor is one of the few factors of intelligence and mathematical ability which has been found to differ between sexes. This factor has not been found for girls, but it has been found for boys. The number factor involves rule inference and computation. Girls, it appears, solve computational problems verbally. It is unclear how rule inference problems or problems involving analysis might be solved by girls. It appears that performance in this area of problem solving is lower than that of boys.

Unique factors. Memory, inductive reasoning and induction have been identified as factors of intelligence. There is no evidence to suggest the nature of the relation between the inductive reasoning and induction factors identified. Similarly, there is no evidence to suggest the nature of their relations with either mathematical ability or achievement. There is evidence which suggests that memory is not a critical factor in mathematical achievement.

Algebraic manipulation and adaptability to a new task have been found to be related to mathematics achievement. There is no evidence to suggest the nature of their relationship with intelligence. No sex differences have been found on any of these factors.

Mathematics achievement. Differences between boys and girls in mathematical achievement do not emerge until the late elementary, junior high and early high school years. Although no consistent evidence is available on the differences at the high school level, one would assume that the trends identified would continue. The differences in mathematical achievement are a difference in kind. Girls tend to do better in computation; boys tend to do better in comprehension, analysis and application.

Differences between the sexes in intelligence and mathematical ability may explain some of these mathematics achievement differences. First, differences between the sexes in mathematics ability and intelligence tend

to appear at the same time in the developmental sequence as do mathematics achievement differences (i.e., at the onset of puberty). The types of intellectual and mathematical ability differences between the sexes which are found to occur could, in part, explain the mathematical achievement differences found between the sexes.

For example, the more frequent loadings for boys on items which involve analysis parallels the requirements involved in the solution of some of the higher cognitive level mathematics items. The higher loadings for girls on verbal items and the use by girls of verbal ability to solve problems involving computation may account for higher achievement in this area.

Conclusions

Evidence suggests that mathematical ability and intelligence are strongly and positively related. The nature of the relation is summarized in Figure 1. Mathematical ability and intelligence are also strongly and directly related to mathematical achievement. Some factors of intelligence and ability are more important than others in explaining mathematical achievement differences in the sexes. The most important factorial differences are in verbal ability and number ability. The superior verbal ability of girls may account for their higher achievement in computation. The superior number (analytic) ability of boys may account for their higher achievement on the higher level cognitive items.

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MATHEMATICS, SPATIAL ABILITY AND THE SEXES

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A paper about mathematics and spatial ability confronts two dilemmas: (1) because there is too much information on spatial ability, it is impossible to review the literature adequately, and (2) there is too little information about spatial ability and mathematics to come to any final conclusion about their relationship. In spite of these dilemmas however, this paper will give an overview of sex differences in spatial ability and suggest how mathematics and spatial ability might be related. Also some researchable hypotheses about how differential spatial ability may effect mathematics achievement will be suggested.

Spatial Ability and the Sexes

Male superiority over females in tasks measuring all kinds of spatial ability has been an accepted truism documented by many authors (Garai and Scheinfeld, 1968; Maccoby, 1966; Kagan and Kogan, 1970; Kogan, 1972; Sherman, 1967; Anastasi, 1958; Tyler, 1956). However, as with many beliefs that have been held about females, new evidence and new interpretations are suggesting that this truism may not be universally true. Eleanor Maccoby, a highly respected psychologist at Stanford University is a pioneer in summarizing the literature in sex differences--particularly in intellectual ability. It is informative to trace her statements about sex differences in spatial ability. In 1966 she states: "by the early school years boys consistently do better on spatial tasks, and this difference continues through the high school and college years", (Maccoby, 1966, p.26). In 1972 Maccoby and Jacklin stated that "spatial ability continues to be the area (i.e., intellectual area) in which the strongest and most significant sex differences are found. Sex differences remain minimal and inconsistent until approximately the age of 10 or 11, when the superiority of boys becomes consistent on a wide range of populations and tests." (Maccoby and Jacklin, 1972, p.41). In 1974 Maccoby and Jacklin sharply restricted their conclusion of male superiority on all types of spatial tasks and moved the age of appearance to adolescence. They said: "Male superiority on visual-spatial tasks is fairly consistently found in adolescence and adulthood, but not in childhood." (Maccoby and Jacklin, 1974, p.351). In 1966 Maccoby believed that males were superior to females in all types of spatial ability starting with the early school years. By 1974--8 years later--she had changed her conclusion to male superiority in only one type of spatial ability, i.e., spatial visualization, with the differences not appearing until adolescence. The accepting by Maccoby in 1974 of a more precise and definitive view of spatial ability appears to explain this remarkable change in belief.

In her earlier writing Maccoby, with her major concern limited to sex differences, treated spatial ability as a unitary ability. This was in conflict with others who were defining and describing spatial ability

(Smith, 1964; Kagan and Kogan, 1970; Werdelin, 1961). The consensus of this writing is that spatial ability is not a unitary factor but is composed of two or more factors, the number and names of these factors differing with various authors (Smith, 1964). Adding to this broader definition of spatial ability was Sherman (1967) who demonstrated that tests supposedly of other abilities, e.g., the Rod and Frame Test and the Embedded Figures Test, included a large spatial component. As the definition of spatial abilities became more precise to those who were writing about sex differences, the literature indicated that males are not superior to females in all spatial abilities and that the differences in male/female performances do not appear at an early age. (See Maccoby and Jacklin, 1974 for a complete review.) However, on tasks measuring one kind of spatial ability, i.e., spatial visualization, data from a variety of sources indicate that males perform at a higher level than females starting about at adolescence and continuing throughout high school and adulthood.

It is interesting to note that sex difference in performance on spatial tasks, broadly defined, does not appear in all cultures. Kabanova-Meller (1970) reports that sex differences on spatial tasks do not exist between Russian boys and girls in Grades 4, 5 and 6, although as is usual in the Russian literature, little empirical data was reported to substantiate this belief. Berry (1966) and Kleinfeld (1973) report that while Eskimos appear to have highly developed spatial skills, no difference is found between spatial ability of male and female Eskimos.

Relationship of Spatial Visualization Ability to the Learning of Mathematics

Spatial visualization involves visual imagery of objects and movement or change in the objects themselves or change in their properties. In other words objects or their properties must be manipulated in one's "mind's eye"--or mentally. In Figure 1, one item from a widely used test of spatial visualization is shown.

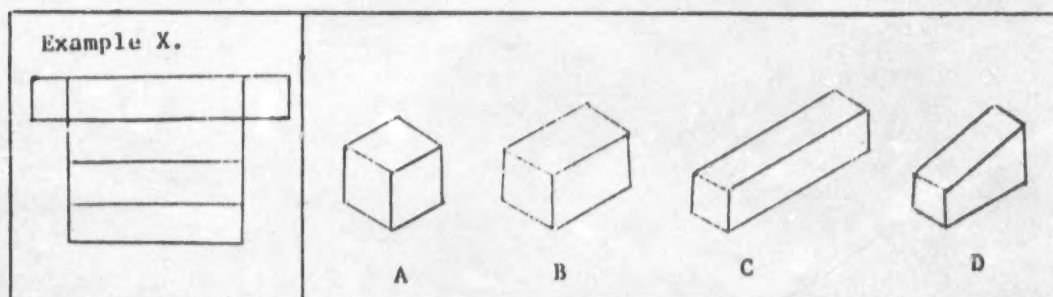
The relationship between mathematics and spatial visualization ability is logically evident. In mathematical terms spatial visualization requires that objects be (mentally) rotated, reflected or translated. These are important ideas in geometry. In fact James and James (1968, p.162) in defining geometry as "the science that treats of the shape and size of things...the study of invariant properties of given elements under specified groups of transformations" are describing accurately most conditions which are met by items on spatial visualization tests.

Many mathematicians believe that all of mathematical thought involves geometrical ideas, as the total discipline of mathematics can be defined as the language for describing those aspects of the world which can be stated in terms of "configurations" (Bronowski, 1947). Meserve (1973, p.249) believes that each person who makes extensive use of all areas of

Figure 1

Space Relations Test of Spatial Visualization¹

This test consists of 60 patterns which can be folded into figures. To the right of each pattern there are four figures. You are to decide which one of these figures can be made from the pattern shown. The pattern always shows the outside of the figure. Here is an example:



¹From: Bennett, G. K., Seashore, H.G., and Wesman, A. G. Differential Aptitude Test: Space Relations, Form T. The Psychological Corporation, NY, 1972. Reproduced by permission. Copyright 1947, © 1961, 1962, 1972 by The Psychological Corporation, New York, NY. All rights reserved.

of mathematics uses the modes of thought of geometry at every turn and that "even the most abstract geometrical thinking must retain some link, however attenuated, with spatial intuition." In the Russian literature, mathematics and spatial abilities are regarded as inseparable (Kabanova-Meller, 1970). Therefore, if spatial visualization items are geometrical in character and if mathematical thought involves geometrical ideas, spatial visualization and mathematics are inseparably intertwined.

Not only are spatial visualization components an integral part of the structure of mathematics, spatial representations are being increasingly included in the teaching of mathematics (e.g., the Piagetian conservation tasks, which are becoming a part of many preschool programs and involve focusing on correct spatial attributes before quantity, length, and volume are conserved). Most concrete and pictorial representations of arithmetical, geometrical and algebraic ideas appear to be heavily reliant on spatial attributes. The number line, which is used extensively to represent whole numbers and operations on them, is a spatial representation. Commutativity of multiplication illustrated by turning an array 90 degrees, involves a direct spatial visualization skill. Many other examples could be cited.

Although the relationship of mathematical ability and spatial visualization ability appears logical, empirical data confirming a positive relationship is less clear. Many factor analytic studies have explored this relationship and several authors have reviewed the literature. Some investigators have definitely concluded that spatial ability and mathematics ability are not related. In 1967, Very (p.172) concluded "research on spatial ability has failed to produce any significant correlation of (the spatial factor) with any facet of mathematics performance." Fruchter (1954, p.2) stated that "spatial ability is unrelated to academic performance with the possible exception of a few very specialized courses such as engineering drawing." Smith (1964, p.127, p.68) concluded that although "there are several studies which indicate consistently that spatial ability is important in tests which are genuinely mathematical as distinct from those which involve purely mechanical or computational processes . . . the question whether the mathematical ability is dependent on the visual factor (or factors) has not been definitely answered."

Even in the specialized mathematical area of geometry where one would expect to find the strongest relationship, empirical findings do not indicate clearly that the two are related. Lim concluded in 1962 after a thorough review of relevant literature "unfortunately the evidence for a relationship between geometric ability and spatial visualization remains inconsistent and unreliable." Werdelin (1961, p.39) also was not willing to definitely conclude that empirical data indicate that spatial visualization ability and geometry ability are related. However, he felt that "there is strong pedagogical reason to believe in a connection between the ability to visualize and geometric ability."

Other authors feel that data indicate a positive relationship. In 1951, Guilford, Green and Christensen concluded that spatial visualization ability helped in solving mathematics problems. French (1951, 1955) also

showed that successful achievement in mathematics depends to some extent on use of spatial visualization skills. In a recent review Aiken (1973, p.406-7) concluded that spatial-perceptual ability was one of the "most salient" mathematical factors extracted in various investigations. Obviously, the relationship between learning in mathematics and spatial ability is not clear and the need for more data is great.

Even less is known about the effect that differential spatial visualization ability has on the mathematics learning of boys and girls. Indication that the relationship between the learning of mathematics and spatial visualization ability is an important consideration, is the paralleling of development of sex differences in favor of males in mathematics achievement and spatial visualization ability. No significant sex differences in either mathematics achievement or spatial visualization task performance have been consistently reported in subjects of 4-8 years of age. Sex differences in performance on spatial visualization tasks become more pronounced between upper elementary years and the last year of high school and the differences show a pronounced increase during this time-span (Maccoby and Jacklin, 1974). It is also commonly believed that sex differences in mathematical achievement appear during this time span and also show a pronounced increase (Fennema, 1974).

It appears reasonable, therefore, to hypothesize that since there is a concurrent developmental trend and since tests of spatial visualization ability contain many of the same elements contained in mathematics, the two might be related. Perhaps less adequate spatial visualization ability may partially explain girls inferior performance in mathematics. However, there are no data available which enables one to accept or to reject this hypothesis. In addition to the need for correlational information, several additional areas when investigated, may provide important insight into the relationship between mathematics learning, spatial visualization ability and the learning of mathematics by boys and girls.

What is the effect of spatial ability on mathematical learning at various developmental levels?

Smith (1964) has hypothesized that while spatial ability may not be related to mathematics ability at beginning stages of mathematics learning, advanced mathematics learning increasingly depends upon spatial ability. It would appear that this hypothesis was made after surveying a number of studies which used high school or college students as subjects and relatively sophisticated mathematical ideas as criterion measures. Little or no data were presented from studies with younger learners. However, in 1964 one could have built a strong argument that logically supported the idea that spatial ability was not related to mathematics ability at beginning stages of mathematics learning. Little or no geometry was taught at the pre-high school level and most pre-high school mathematics tests would not have included geometrical items. Such tests would have focused primarily on arithmetical/computational ideas. Computation has been found to be negatively correlated with spatial ability (Werdelin, 1958). Therefore, Smith's

hypothesis that spatial ability was not related to mathematical ability at beginning stages of mathematics learning was believable in 1964 because the tests used to measure mathematics achievement probably included few items relevant to spatial ability. These tests reflected the mathematics program of most schools previous to 1964.

However, since 1964, a major change has taken place in most K-12 mathematics curricula. More emphasis is placed on the structure of mathematics and its underlying principles. Geometry has become an integral part of the entire mathematics curriculum. Increasingly mathematics is taught as an interrelated system of ideas. In order to learn new ideas, learners are dependent upon prerequisite ideas in their cognitive structure. Little is known concerning the impact of spatial ability on the acquiring of these prerequisite mathematical ideas in which all later mathematical knowledge is based. It appears to be of the utmost importance.

Developmental psychologists patterned on Piaget have theorized that at different stages of cognitive development certain modes of thought predominate and ideas are added to one's cognitive structure by utilizing actions, symbols which represent those actions, and symbols alone, in somewhat different blends. According to this theory, mental structures are formed by a continual process of accommodation to and assimilation of the environment. This adaptation (accommodation and assimilation) is possible because of the actions performed by the individual upon her/his environment. These actions change in character and progress from overt, sensory actions done almost completely outside the individual; to partially internalized actions which can be done with symbols representing previous actions; to complete abstract thought done entirely with symbols. Thus, development in cognitive growth progresses from the use of physical actions to form schemas to the use of symbols to form schemas, i.e., learners change from a predominant reliance upon physical actions to a predominant reliance upon symbols.

Mathematical educators have increasingly accepted this theory of cognitive development and have translated it into educational practice by an increased emphasis upon the instructional use of three modes to represent mathematical ideas, i.e., concrete (enactive), pictorial (ikonic), and symbolic. Such a belief suggests that the blend of the usage of these representational modes should reflect the cognitive developmental level of the learner. Particularly at early stages of mathematical learning it is important to provide learners with concrete representations of mathematical ideas while symbols assume increasing importance as learners mature and mathematical ideas become more complex.

As was illustrated earlier, most concrete and pictorial representations of mathematical ideas include spatial attributes, some of which are relevant to the mathematical idea being taught and some of which are not. Since the only way to add simple mathematical ideas to one's cognitive structure at early developmental levels is by interaction with concrete or pictorial materials which represent those ideas, and since those representations depend heavily on spatial attributes, if for some reason one is hampered in perception of those spatial attributes then one is hampered in learning those

early mathematical ideas. Without knowledge of these ideas, it is impossible to learn advanced mathematics. Therefore, spatial visualization ability appears to be of utmost importance at early stages of learning.

Sherman (1967) has suggested that boys outperform girls on spatial tasks because they participate voluntarily in more spatially oriented activities. Girls learn to read more easily than do boys. Because of ease of use of symbols (i.e., reading) do girls voluntarily, or by encouragement, rely more heavily on symbols to learn mathematics rather than using concrete or pictorial representations? If so, perhaps inadequate usage of spatial representations may hamper both the development of their spatial ability and ability to do well in more advanced mathematical learning.

No data is available to give insight into this question. Empirical data from studies dealing with the use of various representational modes are not conclusive even about the value of concrete and pictorial representations and as far as this author knows no study has included spatial visualization ability as a control factor. Certainly, more data are needed.

What is the interaction effect of other abilities and spatial visualization ability on achievement in mathematics?

Werdelin (1961) showed that girls were able to prove verbal theorems better than boys but were less able to translate words into figural images and then to transform those images in a directed way. It has been another accepted truism that females' verbal ability is more highly developed than males. (Maccoby and Jacklin (1974) believe this is still true.) Does the development of verbal ability in some way interfere with development of spatial ability? Werdelin (1958) in a factor analytic study found one spatial visual factor in high school students which was related to a factor he called a mathematical reasoning factor. Interestingly, he found the correlation between his visual factor and a numerical (or computational) factor was negative. Females often score higher on tests of computation than do males. Perhaps higher development of numerical or computational ability interfered with development of spatial-visualization ability. Both of these questions appear related to the earlier one of the impact of spatial ability on early mathematical learning. Does facility with symbols--computational or verbal--interfere with development of spatial ability?

What is the effect of possessing a greater variety of well developed abilities on mathematics learning?

Harris and Harris (1972), Werdelin (1958), and Very (1967) have shown a larger number of space factors for males than for females. Werdelin (1961) concluded that if one could attack a problem either verbally or spatially, one would be more apt to be able to solve it, and his data showed that boys were superior on items which measured the ability to comprehend the organization of a visual figure and to reorganize. Where items could be solved by verbal means and did not require that the problems be translated into a mental figure, no sex differences were found. Perhaps because males have developed more abilities than have females they are enabled to attack mathematical problems in a variety of ways and thus are able to score higher on mathematical achievement tests.

What sex differences in mathematics achievement would be found if spatial ability were not a factor?

Tittle (1973) has shown that many tests, commonly used to measure achievement, are sexually biased. Certainly if a mathematics test contains many items that require spatial ability to solve, there is a possibility that girls will not do as well as will boys. It would be very interesting to construct a test that had little or no spatial content in it and compare the sexes on achievement. Perhaps no differences will be found if the test content is controlled in the spatial area. On the other hand, spatial visualization may be such an integral part of higher mathematical thinking that eliminating spatial aspects of mathematics tests too narrowly restricts the area of mathematical thinking. This aspect should be investigated.

Implications and Directions for Further Research

Data related to the questions found in this paper will be helpful. Also needed are data which give insight into how spatial visualization ability is developed. Several investigators (Kleinfeld, 1973) have suggested that spatial ability is as important as are other abilities which have received extensive attention in the schools, i.e., verbal ability. Certainly a plethora of abilities will be more effective in dealing with modern day society than will one, so this appears reasonable. Although the main concern of this paper was to explore one facet of why girls achieve at lower mathematical levels than do boys, it is hoped also that one of the outcomes will be an increased awareness of a specialized ability that has received inadequate attention from mathematics educators in recent years. Hopefully, more data will be forthcoming in this important area.

It is tempting to look for a simplistic explanation for sex differences in mathematical achievement. If one says that such differences are the result of differences in spatial visualization ability, one has such a simplistic explanation which is totally inadequate. This paper by no means suggests that sex differences in spatial visualization is the only factor contributing to sex differences in mathematics achievement. Other possible factors include the hypothesis that fewer females than males inherit a gene for quantitative reasoning (Stafford, 1972); stereotyping of mathematics as a male domain; lack of encouragement of females by parents and peers; and lack of clearly perceived vocational plans for females which would include the use of mathematics. Nonetheless, spatial visualization ability is one factor which may contribute to mathematics achievement and the relationship warrants further investigation.

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CONCLUDING STATEMENT

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Mathematics educators have often believed that girls achieve at lower levels in mathematics than do boys. Although this belief has generated no action to improve the learning of mathematics by girls, careful feeding of the literature of the past decade or so reveals that this belief is not valid in all situations. In 1974 Fennema concluded reviewing the literature that "no significant differences between boys' and girls' mathematics achievement were found before boys and girls entered elementary school or during early elementary years. In upper elementary and early high school years significant differences were not always apparent. However, when significant differences did appear they were more apt to be in the boys' favor when higher-level cognitive tasks were being measured and in the girls' favor when lower-level cognitive tasks were being measured." No conclusion could be reached concerning high school learners. (Fennema, 1974, 136-137.) Maccoby and Jacklin made a stronger statement when they concluded that one sex difference that is fairly well established is "that boys excel in mathematics ability." (Maccoby and Jacklin, 1974, 352.) While this conclusion appears to be based on an incomplete review of the literature and is ambiguous because no distinction is made between ability and achievement, at the very least such a strong statement points up the need for in-depth analysis of the learning of mathematics by girls.

Even though no definite conclusion can be reached at this time about the comparative levels of learning mathematics by the sexes, it is clear that, starting at about the tenth grade and continuing throughout all post high school education, girls increasingly chose not to study mathematics. Economic as well as moral reasons compel mathematics educators to be concerned with this problem. If mathematics is important for boys, it is equally important for girls. However, before a cry for change in the mathematical education of girls can be made, it is important to take stock of what is known about mathematics learning by boys and girls. These papers have suggested hypotheses that, when data is available, will provide some insight concerning comparative mathematics learning by the sexes, and why they are unequal in their studying of mathematics.

The main factors related to sex differences in the learning of mathematics which are emphasized in this set of papers are intellectual factors. Largely omitted is the large set of social/cultural factors which effect the learning of mathematics as it is related to the development of one's sex role identity. A discussion of these highly important factors was omitted because, while there is a large body of knowledge about factors which effect the sexes differentially in achievement motivation in general, there is a paucity of data which deal explicitly with the learning of mathematics and the development of sex role identity. The reader should not assume that the omission of these factors reflects the lack of their impact on the learning of mathematics. On the contrary, it is hoped that lack of discussion of such societal influences will stimulate studies that will give direct information on the learning of mathematics and sex role

development. The influence of such factors as stereotyping of mathematics as a male domain, differential expectations of significant others (teachers, parents, peers) toward the person as a learner of mathematics and attitude toward success in mathematics literally demand exploration.

It is a changing world. The Women's Liberation Movement and economic realities have combined to make the role of females different in 1975 than it has been in the past. This "role" of females (or in reality "roles") includes the learning of mathematics and it is risky to say that any conclusions reached in these papers on the basis of previous studies would necessarily be the conclusions reached if new data were available. Therefore, these papers are seen as a starting point from which concerned researchers and teachers can start to equalize the studying and learning of mathematics by males and females. Certainly as researchers and teachers become more aware, and as a result more concerned, about the "drop out" rate of girls in the studying of mathematics and perhaps the less adequate learning of mathematics by girls new insight will be generated into the learning of mathematics by both sexes.

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